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IMPROVED PROPAGATION MODELS
IRREGULAR MEDIA

FINAL REPORT

by Charles L. Rino

30 June, 1991

U. S. ARMY RESEARCH OFFICE

Grant Number: DAALO3-89-C-0011

Vista Research, Inc.
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Mountain View, CA 94042

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ABSTRACT

This final report summarizes the most important results obtained from the research supported under the Army Research Office Grant No. DAALO3-89-C-0011. We have completed work that was initiated under a previous ARO contract (DAAL03-87-C001) to develop a consistent and rigorously correct formalism that can be applied to both discrete and continuous random media. That work has led to a new analytic/computational method, which we call the mutual interaction method (MIM). The method has been successfully applied to problems involving a discrete scatterer near a random surface. It also forms the basis for highly efficient computational algorithms for numerical simulations.

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I Statement of the Problem

In continuous random media, one invariably uses the parabolic approximation to the wave equation. Thus, the development of moment equations that characterize the random field proceeds from a model that excludes a priori wide-angle scattering and backscatter. While attempts have been made to rectify both limitations, the formulations used are intractable or inconsistent. It is desirable to use a formulation that accommodates backscatter and wideangle scatter at the outset.

In discrete random media, the formalism developed by Flody, Lax, and Twersky is more often used. The problem development is setup so that a self-consistent computation of the complete multiple scattering interactions amoung all the scatters is accommodated. It is known that self-consistent interaction computations can be set up as solutions to differential equations as well as implicit summations of all interactions (diagram methods). Whereas the continuous media problem generally proceed from a system of restricted differential equations, the discrete problem more often proceeds an exact diagram system. It is desirable to use a common formulation that preservs the self-consistent interaction fields but can be transformed to diagram form.

We believe that our current work has achieved this goal and provides some new insights into the structure of scattering problems.

II Summary of Principal Results

Completing work that was initiated under a previous ARO contract (DAAL03-87-C-002), we have developed a consistent formalism akin to the multiple-phase-screen model that is rigorously correct for both discrete and continuous random media. For example, the mutual interaction form of the propagation equations for continuous random media can be written as follows:

$$\frac{\partial \hat{\psi}^{+}(\mathbf{K};z)}{\partial z} = ikg\,\hat{\psi}^{+}(\mathbf{K};z) + \frac{ik}{2g(K)}\iint \widehat{\delta\epsilon}(\mathbf{K}-\mathbf{K}';z)\,\hat{\psi}(\mathbf{K}';z)\frac{d\mathbf{K}'}{(2\pi)^2} \tag{1}$$

$$-\frac{\partial \hat{\psi}^{-}(\mathbf{K};z)}{\partial z} = ikg\,\hat{\psi}^{-}(\mathbf{K};z) + \frac{ik}{2g(K)}\iint \widehat{\delta\epsilon}(\mathbf{K}-\mathbf{K}';z)\,\hat{\psi}(\mathbf{K}';z)\frac{d\mathbf{K}'}{(2\pi)^2} \tag{2}$$

The integral term involves the total wave field

$$\hat{\psi}(\mathbf{K};z) = \hat{\psi}^{+}(\mathbf{K};z) + \hat{\psi}^{-}(\mathbf{K};z), \tag{3}$$

and $\widehat{\delta\epsilon}(\mathbf{K};z)$ is the spatial Fourier transform of the relative permittivity variation. Insofar as we know, these equations have not been used previously.

In [1], we describe the results of a detailed study of the conditions under which a hirearchy of moment equations can be derived from (1) and (2). We show that two distinct approximations approximations are involved. The first approximation requires small local perturbations as a sufficient condition for evaluating the functional derivatives that appear in the Novikov-Furutsu (NF) theorem. This assumption alone permits the development of a closed hirearchy of differential equations for the signal moments. The second approximation requires that over short distances the spectral components propagate like plane waves in a homogeneous medium. Under the first and second approximations, the most general form of the integral equations for the second order-signal moments that depend only on the spectral density function of the permittivity fluctuations can be developed. Upon simplifying these equations by using the narrow-angle scatter approximation, we reconfirmed an earlier finding, namely that backscatter enhancements are negligibly small in continuous random media under the usual conditions of the Markov approximation.

In earlier work [2], we performed a comparative analysis of solutions to the one-dimensional form of the wave equation. The method of invariant imbedding provides a rigorous alternative solution at the level of the first approximation. Thus, the second approximation can be tested. We found that for the first-order moments of the field, the Markov solution shows little error; however, the total flux within the medium admits a systematically increasing error as the portion of the incident flux that has been backscattered increases. The invariant imbedding method does not resolve the internal flux into directed wave fields, but for a lossless slab of sufficient depth, all of the incident intensity will be backscattered. The breakdown of the second approximation is of practical concern for sound and EM propagation in highly inhomogeneous media such as soil or mud. Similar limitations most likely will apply in nonsparse discrete random media.

Unfortunately, configurational averaging in discrete random media has not produced rigorous solutions at the level of the invariant imbedding solution. Thus, in [3], we assumed that the fields at the boundary planes of the nth slab are uncorrelated with the scatterers within the slab. With this assumption, it is possible to derive a closed hierarchy moment equations for the vector fields. We showed that the characteristic equation for the mean field propagating in a statistically homogeneous medium has four eigenvalues corresponding to orthogonally polarized waves propagating in the $\pm z$ directions. For the scalar wave equation, the paired eigenvalues are given as

$$\lambda = \pm\left(\Delta \pm \sqrt{(1+\bar{\sigma}^{++})(1+\bar{\sigma}^{--}) - \bar{\sigma}^{+-}\bar{\sigma}^{-+} + \Delta^2}\right), \tag{4}$$

where

$$\Delta = (\bar{\sigma}^{++} - \bar{\sigma}^{--})/2, \tag{5}$$

and $\bar{\sigma}^{\pm\mp}$ represents the ensemble average of the corresponding scattering function for single slab but normalized by the slab thickness. When $\bar{\sigma}^{++} = \bar{\sigma}^{--} = \bar{\sigma}^{f}$ and $\bar{\sigma}^{+-} = \bar{\sigma}^{-+} = \sigma^{b}$, the result agrees with a well-known result derived by Twersky, which is commonly used to estimate the extinction effects of backscatter.

The uncorrelated field hypothesis appears to be valid for sparse media, but its limitations presently are uncertain. Thus, we have not yet extended the computations to higher-order moments.

We have applied the MIM method to practical problems involving an object scattering near a highly rough surface. Under a separate contract, we applied the method to investigate the hypothesis that enhanced acoustic surface reverberation in the ocean is caused by subsurface bubble clouds [4]. These computations provide a good example of the comparative ease with which the results from numerical simulations and known scattering functions can be combined via MIM. A surface-scatter simulation was developed for dynamically evolving nonlinear ocean surface realizations as one input to the MIM computation [5]. The bubble cloud was modeled as a cylindrical void with known scattering characteristics, although analytic continuation must be used to extend the Bessel series to accommodate evanescent waves. The general problem of a particle scattering near a rough surface is reviewed in [6].

The scattering problem is simplified considerably when the integration implied by the @ operator can be replaced by a simple product. In [7], we describe a class of scattering objects for which the scattering operator can be evaluated algebraically. This is important, because the general scattering interaction of a wave field with a particle requires the equivalent of an integral operation whether formulated in the spatial or Fourier domains. The T-matrix method effectively converts the integral to an infinite series of spherical-harmonic modes. A finite subset of the coefficients are manipulated with the T-matrix approach. For objects that do not deviate strongly from spherosity, this method is efficient; however, for a broader class of scattering objects, direct computation of the scattering function as described in [8] may be more efficient.

Finally, although it was not formally part of our current ARO contract, we used our surface-scatter code to study the statistics of backscatter enhancements. Our results confirmed earlier findings that speckle statistics remain gaussian distributed through the region of the backscatter enhancement [9]. We have also simulated backscatter enhancements for highly nongaussian surfaces at near-grazing incidence. Results from our simulation codes have been made available to other ARO researchers.

Some unpublished work as yet unpublished is described in more detail in the next section.

III The Relation of MIM to Order N Algorithms

III.1 Background

Wayne Chew [10] has described a recursive algorithm that evidently can achieve N^2 computational efficiency for a system of N mutually interacting scatterers. The scheme has been compared to $O(N^3)$ method-of-moments solvers to verify that it is indeed solving comparable problems with substantially improved efficiency [11]. More recently, an even faster implementation of the algorithm has been described [12]. The latter scheme is structurally similar to the mutual interaction method [3], which we believe to be capable of similar efficiencies. The relation is difficult to see, however, because Chew's method uses the T-matrix formalism; however, Chew's method can be developed more generally.

Consider a system of N isotropic scatterers. The structure of the solution will show how the computational efficiency is achieved and how to reformulate the method in terms of other scattering operators [7]. For an incident wave $\psi(\mathbf{r})$ scattering from a system of N scatterers, the total field at any exterior point $\mathbf{r}$ can be written as

$$\psi^N(\mathbf{r}) = \psi(\mathbf{r}) + \sum_{j=1}^{N} \psi_j^s(\mathbf{r}), \tag{6}$$

where ψ_j^s represents the field scattered by the jth scatterer in the presence of all other scatterers. This field can be related linearly to the total wave field incident upon the scatterer, ϕ_N^j, as follows:

$$\psi_j^s(\mathbf{r}) = \iiint_{V_j} S_j(\mathbf{r}, \mathbf{r}')\, \phi_N^j(\mathbf{r}')\, d\mathbf{r}', \tag{7}$$

where $S_j(\mathbf{r}, \mathbf{r}')$ depends only on the attributes of the jth scatterer and the integration is carried out over its volume V_j. The total field incident upon the jth scatterer admits the representation

$$\phi_N^j(\mathbf{r}) = \psi(\mathbf{r}) + \sum_{\substack{n=1 \\ n\neq j}}^{N} \iiint_{V_n} S_n(\mathbf{r}, \mathbf{r}')\phi_N^n(\mathbf{r}')\, d\mathbf{r}'. \tag{8}$$

The system of N equations implied by (8) can, in principle, be solved for the unknown fields $\phi_N^j(\mathbf{r})$, which need be determined only within the confines of each scatterer where $S_j(\mathbf{r}, \mathbf{r}')$ is finite. The formal solution to (8) can be written as

$$\phi_N^j = \mathbf{T}_N^j \boldsymbol{\Psi}, \tag{9}$$

where $\mathbf{T}_N^j$ is the jth row of the $N \times N$ matrix operator $\underline{\mathbf{T}}_N$ and $\boldsymbol{\Psi}$ is a column vector whose nth element consists of the incident field evaluated in the volume of the nth scatterer. In terms of $\underline{\mathbf{T}}_N$, (6) can be written as

$$\psi^N(\mathbf{r}) = \psi(\mathbf{r}) + \sum_{j=1}^{N} \iiint S_j(\mathbf{r},\mathbf{r}')\, \mathbf{T}_N^j \boldsymbol{\Psi}\, dr'. \tag{10}$$

As a practical matter, the problem is solved when the fields ϕ_N^j are determined. As with any linear system of equations more computation is required to determine $\underline{\mathbf{T}}_N$ itself; moreover, the scattered field must be evaluated in the region of interest. This formalism is attributed to Foldy, Lax, and Twersky (FLT).

III.2 Isotropic Scatterers

An object located at $\mathbf{r}_j$ is called a point-like isotropic scatterer if its scattering function S_j has the following property:

$$\iiint_{V_j} S_j(\mathbf{r},\mathbf{r}')\psi_N^j(\mathbf{r}')\, d\mathbf{r}' = h_j \psi_N^j(\mathbf{r}_j)\, G(\mathbf{r},\mathbf{r}_j), \tag{11}$$

In (11), the right hand side is the scattered wave observed at $\mathbf{r}$, ψ is an arbitrary incident wave, h_j is a constant characterizing the object's scattering strength, and G is the outgoing Green's function defined by

$$G(\mathbf{r},\mathbf{r}') \equiv \frac{\exp\{ik|\mathbf{r}-\mathbf{r}'|\}}{4\pi|\mathbf{r}-\mathbf{r}'|}. \tag{12}$$

Consider a rightward traveling incident wave $\psi(\mathbf{r})$ multiply scattered by N point-like isotropic scatterers. The FLT equations simplify to

$$\phi^N(\mathbf{r}) = \psi(\mathbf{r}) + \sum_{j=1}^{N} h_j \phi_N^j G(\mathbf{r},\mathbf{r}_j) \tag{13}$$

and

$$\phi_N^j = \psi(\mathbf{r}_j) + \sum_{\substack{n=1 \\ n\neq j}}^{N} h_n \phi_N^n G_{jn}. \tag{14}$$

where $G_{jn} \equiv G(\mathbf{r}_j, \mathbf{r}_n)$. The latter equation can be written in matrix form as

$$\begin{bmatrix} 1 & -h_2G_{12} & -h_3G_{13} & . & . & -h_NG_{1N} \\ -h_1G_{21} & 1 & -h_3G_{23} & . & . & -h_NG_{2N} \\ -h_1G_{31} & -h_2G_{32} & 1 & . & . & -h_NG_{3N} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ -h_1G_{N1} & -h_2G_{N2} & . & . & . & 1 \end{bmatrix} \begin{bmatrix} \phi_N^1 \\ \phi_N^2 \\ \phi_N^3 \\ . \\ . \\ . \\ \phi_N^N \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ . \\ . \\ . \\ \psi_N \end{bmatrix} \tag{15}$$

If we let $\mathbf{T}_N$ represent the inverse of the matrix in (15), the formal solution is

$$\boldsymbol{\Phi}_N = \mathbf{T}_N \boldsymbol{\Psi}. \tag{16}$$

We could equally well work in terms of the scattered fields $\psi_N^j = h_j \phi_N^j$. Either way, the direct solution to this system of equations would require $O(N^3)$ operations

III.3 A Fast Algorithm for Isotropic Scatterers

Now consider a system of N isotropic scatterers with individual strength h_j as described in Section III. We will formulate the solution in terms of ψ_N^j, which represents the total scattered field at the location of the jth particle, $\mathbf{r}_j$. In terms of Ψ_N^j, the total scattered field can be written as

$$\psi^N(\mathbf{r}) = \psi(\mathbf{r}) + \sum_{j=1}^{N} G(\mathbf{r}, \mathbf{r}_j)\psi_N^j. \tag{17}$$

Recall that

$$\psi_N^j = h_j \phi_N^j, \tag{18}$$

where ϕ_N^j is the total field incident upon the jth particle. Moreover, $\psi(\mathbf{r})$ is a freely propagating wave field. Thus, if the field is known on a plane, say at $z = z_0$, it can be transformed to any other plane, say at $z = z_j$, by using the propagation operator denoted as P_{j0}.

This property can be used to develop an efficient recursive algorithm for computing ψ_N^j. For a single isolated particle, $\psi_1^j = h_j \phi_1^j$. If two particles are present, it follows that

$$\psi_2^1 = h_1 \left[\psi_1 + G_{12}\psi_2^2\right] \tag{19}$$

$$\psi_2^2 = h_2 \left[\psi_2 + G_{21}\psi_2^1\right] \tag{20}$$

The terms in square brackets represent the direct and scattered wave impinging upon the the particular scatterer. For the propagating wave field,

$$\psi_j = P_{j0}\psi_0. \tag{21}$$

The direct parallel to Chew's [10] development can be seen by the following equivalences:

$$\begin{aligned} f_j &\equiv \psi_2^j & (22)\\ T_{j(1)} &\equiv h_j & (23)\\ \alpha_{ij} &\equiv G_{ij} & (24)\\ \beta_{j0} &\equiv P_{j0} & (25)\\ a &\equiv \psi_0 & (26) \end{aligned}$$

We now substitute (19) into (20) and use the relation $P_{10}\psi_0 = P_{12}P_{20}\psi_0$ to obtain

$$\psi_2^2 = h_2 \frac{[1 + P_{12}G_{21}h_1]}{1 - h_1 h_2 G_{12} G_{21}} P_{20}\psi_0 \tag{27}$$

If we define

$$T_{2(2)} = \frac{h_2[1 + P_{12}G_{21}h_1]}{1 - h_1 h_2 G_{12} G_{21}}, \tag{28}$$

we see that the scattered field at the second scatterer is given in terms of a constant multiplying the incident field at the location of the scatterer, $P_{20}\psi_0$. Substituting (28) into (19) and manipulating the incident field yields the complementary relation for the first scatterer, namely

$$T_{1(2)} = h_1[1 + G_{12}T_{2(2)}P_{21}]. \tag{29}$$

Because $T_{j(1)} = h_j$, we see that the 2-particle T elements are given in terms of the single particle T elements.

For an n-particle system,

$$\psi_n^j = T_{j(n)}\psi_j = T_{j(n)}P_{j0}\psi_0. \tag{30}$$

From (17), it follows that

$$\psi^{n+1} = \psi + \psi_{n+1}^{n+1} + \sum_{j=1}^{n} \psi_{n+1}^j. \tag{31}$$

The form of (31) together with (30) implies that

$$\psi_{n+1}^j = T_{j(n)}\left[P_{j0}\psi_0 + G_{j,n+1}\psi_{n+1}^{n+1}\right], \tag{32}$$

which is the general form of (19). Also, for an $n+1$-particle system,

$$\psi_{n+1}^{n+1} = h_{n+1}\left[P_{n+1,0}\psi_0 + \sum_{j=1}^{n} G_{n+1,j}\psi_{n+1}^{j}\right], \tag{33}$$

which is the generalized form of (20). Proceeding exactly as with the two-particle system, we substitute (32) into (33) and manipulate the result to obtain

$$\psi_{n+1}^{n+1} = h_{n+1}\frac{1+\sum_{j=1}^{n} G_{n+1,j}T_{j(n)}P_{j,n+1}}{1-h_{n+1}\sum_{j=1}^{n} G_{n+1,j}T_{j(n)}G_{j,n+1}}P_{n+1,0}\psi_0, \tag{34}$$

from which it follows that

$$T_{n+1(n+1)} = T_{n+1(1)}\frac{1+\sum_{j=1}^{n} G_{n+1,j}T_{j(n)}P_{j,n+1}}{1-h_{n+1}\sum_{j=1}^{n} G_{n+1,j}T_{j(n)}G_{j,n+1}}. \tag{35}$$

Substituting (35) into (20) and manipulating the incident field completes the recursion:

$$T_{j(n+1)} = T_{j(n)}\left[1 + G_{j,n+1}T_{n+1(n+1)}P_{n+1,j}\right]. \tag{36}$$

Equations (35) and (36) are functionally identical to Chew's [10] equations (17) and (18). Although P_{j0} is an operator that would generally require an FFT to evaluate, the total number of operations required to evaluate (35) and (36) is $O(N^2)$. From this discussion it is clear that the computational efficiency arises by virture of the fact that the incident wavefield is freely propagating and therefore can be mapped from position to position. The scattered fields have the same property outside the scatterers. The T-matrix addition theorem propagates the incident fields (via β_{ij}) and the scattered fields (via α_{ij}) from the location of one particle to another.

We have used plane and spherical-wave propagators, but the procedure is general More important, there are other ways to exploit and improve on these efficiencies within the context of the general operator formalism described in [8]. As Chew has noted, the summation terms in (35) and (36) can be put into a common form, which is the aggregate scattering function for the collection of particles. In terms of the aggregate scattering function, the solution is effectively a succession of two-element scattering systems. We have noted that the Mutial-Interaction-Equations could be solved in this fashion, with improved efficiency. Chew's results show that this conjecture is correct.

III.4 An Example

For the simple case of an incident plane wave (a delta-function in the spatial Fourier domain), the propagation operator is multiplicative, whereby all operations are purely

algebraic. In the T-matrix formalism, this is an $M = 1$ system. We have programmed the recursive algorithm and compared the results to the exact solution for both accuracy and efficiency. With the caveat that the system is not close to being singular, in which case the denominator in (35) and (36) is zero, the errors are negligible. The CPU time for a SUN SPARC 2 computer is shown in the figure. The upper curve is the LU decomposition; the lower curve is the recursive algorithm. The variations from the expected N^3 and N^2 curves is most likely due to the dynamic operating environment. The main point is that for the large system, the computation time is reduce from more than one hour to a few minutes.

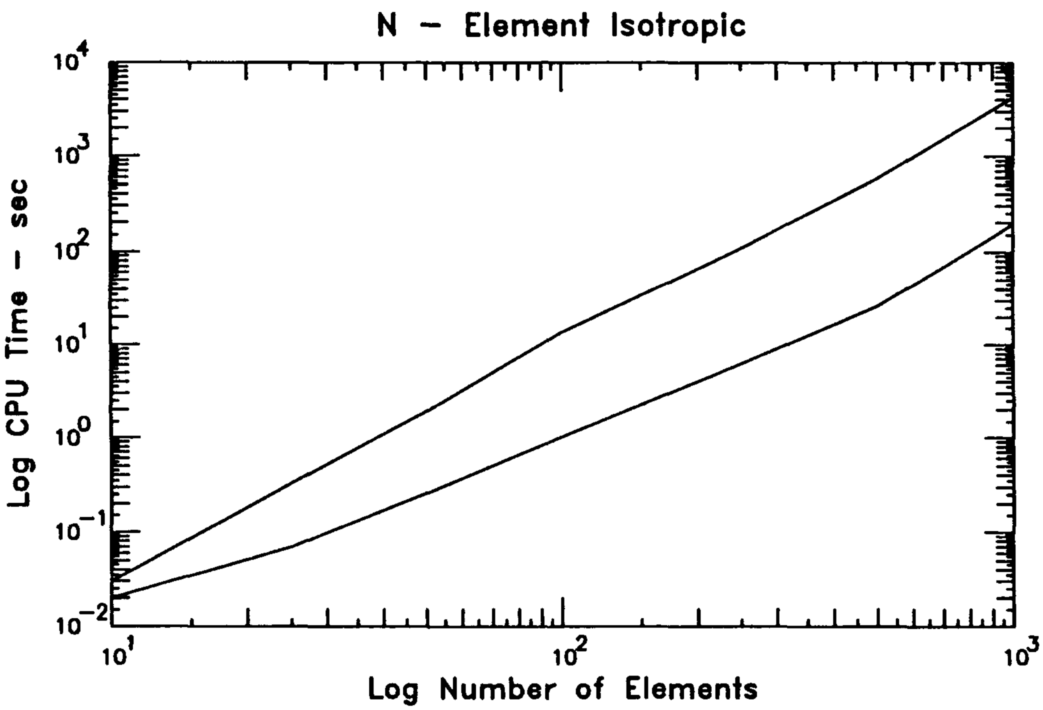

Figure 1: CPU time comparison of direct and recursive solutions.

IV Personnel

During the course of this research effort, the following Vista employees participated in addition to the principal investigator:

Dr. Hoc D. Ngo
Dr. Robert D. Word
Dr. Thomas L. Crystal
Angela Regalia

References

[1] Charles. L. Rino. On propagation in continuous random media. *Waves in Random Media*, 1(1):59–72, 1991.

[2] Charles. L. Rino. A comparison of spectral-domain methods with direct solutions to the one-dimensional wave equation. *J. Electromagnetic Waves and App.*, 5(2):161–173, 1991.

[3] C. L. Rino, H. D. Ngo, and K. A. Haycock. A spectral-domain method for multiple scattering in discrete random media. *IEEE Trans. on Ant. and Prop.*, 38(7):1118–1127, 1990.

[4] Charles L. Rino and Hoc D. Ngo. Numerical simulation of acoustic scatter from subsurface bubble clouds. 1991. (Accepted for publication *J. Acoust. Soc. Am.*).

[5] Charles L. Rino, Thomas L. Crystal, Alan K. Koide, Hoc D. Ngo, and Harold Guthart. Numerical simulation of acoustic and EM scattering from linear and nonlinear ocean surfaces. *Radio Science*, 26(1):51–71, 1991.

[6] Charles L. Rino and Hoc D. Ngo. Scattering from objects near a rough surface. San diego spie proceedings, Vista Research, Inc., 21-26 July 1991.

[7] Hoc D. Ngo and Charles L. Rino. Applications of the mutual interaction method to a general class of two-scatterer systems. 1991. (in preparation).

[8] Hoc D. Ngo and Charles L. Rino. A Green's-function-like approach to multiple scattering. 1991. (in preparation).

[9] M. Nieto-Vesperinas and J. M. Soto-Crespo. Monte carlo calculations of speckle contrast from perfectly conductive rough surfaces. *Optics communications*, 75(3,4), 1990.

[10] W. C. Chew. An n^2 algorithm for the multiple scattering solution of n scatterers. *Microwave and Optical Technology Letters*, 2(11):380–383, 1989.

[11] Y. M. Wang and W. C. Chew. An efficient algorithm for solution of a scattering problem. *Microwave and Optical Technology Letters*, 3(1):102–106, 1990.

[12] W. C. Chew, J. A. Friedrich, and R. Geiger. A multiple scattering solution for the effective permittivity of a sphere mixture. *IEEE Trans. Geoscience and Remote Sensing*, 28(2):207–214, 1990.